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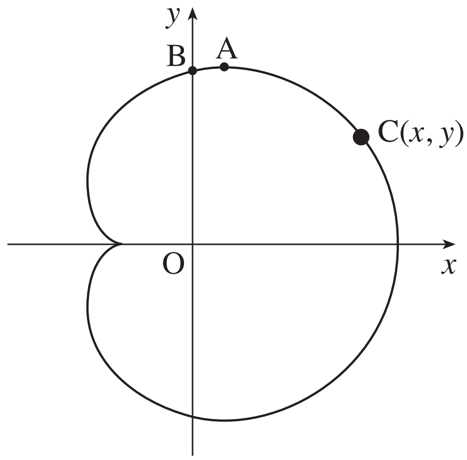

Fig. 8

In a theme park ride, a capsule C moves in a vertical plane (see Fig. 8). With respect to the axes shown, the path of C is modelled by the parametric equations

$$x = 10\cos\theta + 5\cos 2\theta, \quad y = 10\sin\theta + 5\sin 2\theta, \qquad (0 \leqslant \theta < 2\pi),$$

where x and y are in metres.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$.

Verify that $\dfrac{dy}{dx} = 0$ when $\theta = \frac{1}{3}\pi$. Hence find the exact coordinates of the highest point A on the path of C. [6]

(ii) Express $x^2 + y^2$ in terms of θ. Hence show that

$$x^2 + y^2 = 125 + 100\cos\theta. \qquad [4]$$

(iii) Using this result, or otherwise, find the greatest and least distances of C from O. [2]

You are given that, at the point B on the path vertically above O,

$$2\cos^2\theta + 2\cos\theta - 1 = 0.$$

(iv) Using this result, and the result in part (**ii**), find the distance OB. Give your answer to 3 significant figures. [4]

2 Fig. 6 shows the arch ABCD of a bridge.

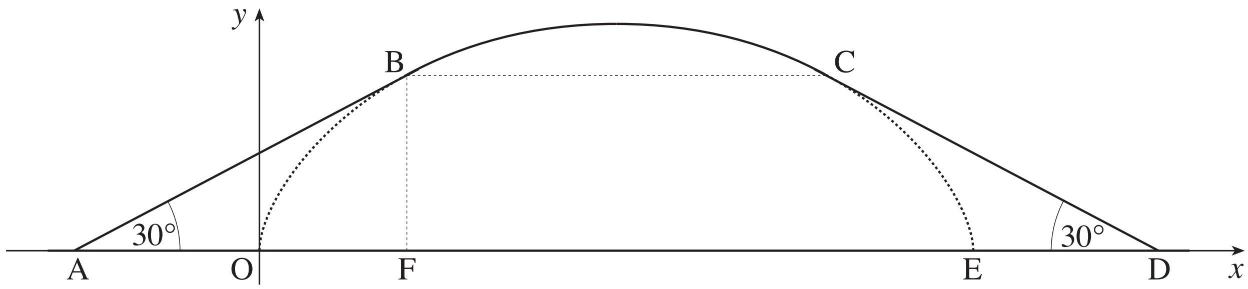

Fig. 6

The section from B to C is part of the curve OBCE with parametric equations

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta) \text{ for } 0 \leqslant \theta \leqslant 2\pi,$$

where a is a constant.

(i) Find, in terms of a,

(A) the length of the straight line OE,

(B) the maximum height of the arch. [4]

(ii) Find $\frac{dy}{dx}$ in terms of θ. [3]

The straight line sections AB and CD are inclined at 30° to the horizontal, and are tangents to the curve at B and C respectively. BC is parallel to the x-axis. BF is parallel to the y-axis.

(iii) Show that at the point B the parameter θ satisfies the equation

$$\sin\theta = \frac{1}{\sqrt{3}}(1 \quad \cos\theta).$$

Verify that $\theta = \frac{2}{3}\pi$ is a solution of this equation.

Hence show that BF $= \frac{3}{2}a$, and find OF in terms of a, giving your answer exactly. [6]

(iv) Find BC and AF in terms of a.

Given that the straight line distance AD is 20 metres, calculate the value of a. [5]

3 A curve has carlesian equation $y^2 - x^2 = 4$.

(i) Verify that

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t},$$

are parametric equations of the curve. [2)

(ii) Show lhat $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t^2+1}$. Hence find the coordinates of the stationary points of the curve. [6]

4 The parametric equations of a curve are

$$x = \sin\theta, \quad y = \sin 2\theta, \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

(i) Find the exact value of the gradient of the curve at the point where $\theta = \frac{1}{6}\pi$. **[4]**

(ii) Show that the cartesian equation of the curve is $y^2 = 4x^2 - 4x^4$. **[3]**

5 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \qquad y = \frac{1-t}{1+2t}.$$

Find t in terms of x. Hence find the cartesian equation of the curve, giving your answer as simply as possible. **[5]**

6 A curve has parametric equations

$$x = e^{2t}, \quad y = \frac{2t}{1+t}.$$

(i) Find the gradient of the curve at the point where $t = 0$. **[6]**

(ii) Find y in terms of x. **[2]**